

in the following manner by the method of triangulation. First setting out from A (see Fig. 1) in whatever direction the character of the ground permits, a base-line  $A d$  is measured with the greatest possible accuracy. At the point A, the angle  $d A e$ , and at the point  $d$  the angle  $A d e$  are observed with a circular instrument. Thus in the triangle  $A d e$ , the adjacent side  $A d$  and the two other parts of the angles being known, the triangle can be computed. Place now in the straight line connecting A and B (in the same meridian) a point C, which can be seen from the points  $d$  and  $e$ ; then we may, by means of the theodolite, measure at  $d$  and  $e$  the angles  $A d c$  and  $A e c$ .<sup>1</sup> Subtract now from these angles the previously observed angles  $A d e$  and  $A e d$ , and we have now found in the second triangle,  $C d e$ , the angles  $d$  and  $e$ . But then, also, the side  $d e$ , as belonging to the first triangle, is known, and thus also the second triangle, and consequently its sides  $C d$  and  $C e$  are known. But if the triangles  $A c d$  and  $A e d$  are known, so are also the triangles  $A d c$  and  $A e c$ ; consequently, also, the common side  $A C$ ; and thus a part of the distance is measured. To obtain the length of the other part  $B C$ , a base  $B h$  will be measured from B, and by operations similar to the above  $B C$  will easily be found. As a test of the accuracy of the measurements, we may connect the first operation, starting from  $C e d$  towards  $f$  and  $d$ , and going on to B, and obtain by means of the agreement of the measured length  $B h$  with the calculated length of  $B h$  as a side of the triangle  $B h f$ , a proof of the accuracy of the measurements of base and angles. Should the length  $A B$  be very great and the intervening ground mountainous, a very great number of small triangles may be required: in which case, though the principle is exactly the same, yet in practice, on account of the numerous measurements necessary under such circumstances, unavoidable errors and inaccuracies will certainly accumulate.

As we have already said, Snellius, in the year 1615, was the first to measure a degree by the method of triangulation. He measured a base line on the plain between Leyden and Sontweronde (316 Rhenish rods and 4 feet long), and by means of connected triangles obtained an arc of the meridian (between Alkmaar and Bergen-op-zoom) of  $1^{\circ} 11' 30''$ . Although Snellius was in possession of an improved instrument (Galileo had already taught the use of the recently-discovered telescope<sup>2</sup> for astronomical purposes), yet his measurements were so inaccurate that he obtained far too small a result (55011 toises for a degree). He soon became convinced of the erroneous nature of his result, and seven years after repeated the operation, measuring in the neighbourhood of Leyden a base-line in the ice. Probably deterred by the multifarious and difficult numerical operations which were at that time connected with the working out of the calculation of this new measurement by means of arithmetic, he did not carry this out, but his successor, Muschenbroek, devoting himself to the execution of this work after revising the triangulation, found 57033 toises as the length of a degree in the Netherlands.

Although the method of triangulation used by Snellius was a great step in advance, yet it was a long time before it became generally adopted; for even in the years 1633 to 1635 a degree-measurement was carried out by Norwood between London and York after the old method. He used an improved instrument (a five-foot sector) and obtained as the difference in latitude of the two places  $2^{\circ} 28'$ , and the length of a degree 57424 toises. Newton, who shortly after began the elaboration of his theory of universal gravitation, did not, at all events, know this result, since he took as the basis of his researches the earlier very inaccurate results as to the dimensions of the earth, and since he found his calculations did not correspond with them he abandoned for a time his theory.

Soon after, Picard, at the instance of the Paris Academy of Sciences, undertook anew a meridian measurement, and that not only after the improved method of Snellius (since he measured all three angles of each triangle, and computed the length of the arc by pieces), but he also gave to the measuring instruments a hitherto unattained accuracy by the addition of a micrometer apparatus.<sup>3</sup> He measured on the meridian of Paris an arc of  $1^{\circ} 22' 55''$ , and finding for the latitude of that place  $49^{\circ} 13'$ ,

<sup>1</sup> There is no necessity for the point  $c$  being taken in a line between A and B, nor any advantage even if it could be done. The angles need not be measured in the way here laid down.

<sup>2</sup> This remark seems to imply that Snellius used a telescope in measuring angles. The application of the telescope to circular instruments was a step taken by Picard.

<sup>3</sup> Picard adapted to his measuring instrument a telescope with cross-wires in its focus; this appears to be the only "micrometer apparatus."

with the, as we now know, wonderfully accurate result of 57060 toises for the length of a degree. When Newton, in 1682, learned the result of Picard's measurement, he resumed his calculations in gravitation, and had the satisfaction, after thoroughly revising his work, of seeing his theory of gravitation established. A few years afterwards he gave to the world his immortal work on the mechanics of the universe. For a short time Picard's dimensions of the earth were accepted as correct and were universally made use of. But while hitherto the measurements had reference alone to the discovery of the size of the earth—for its spherical form was taken as proved—there now began a new epoch in the solution of the second part of the problem—the true figure of the earth. Influenced by the fact that the length of a degree measured at different places on the earth always gave a different result—which could not in all cases be ascribed to inaccurate measurement—Picard had already broached the idea that the earth could not be a true sphere. Soon after, Newton, in his great work, showed, on the supposition that the earth existed originally in a fluid state, that on account of the rotation round the polar axis, the supposed spherical form must be more truly that of an elliptical spheroid, the polar diameter being diminished and the equatorial diameter increased. Shortly after Huyghens was led to the same result; and while Newton by his calculations found the polar diameter to be to the equatorial as 229 to 230, Huyghens, on the basis of less general theories, found the proportion to be 577 to 578. Indeed, although differing somewhat in magnitude (Newton's proportion was then accepted as the more correct), yet, in principle, they both led to the same result, viz., that the earth is flattened at the poles, so that the length of a degree near the poles must be greater than in the neighbourhood of the equator. Moreover, Newton had shown experimentally the flattening at the pole, by rotating a soft clay sphere quickly round its axis, by which it became flattened at its poles.

To this was now added another valuable proof. The French astronomer Richer, in the prosecution of his observations at Cayenne, found to his astonishment that his pendulum, which beat seconds in Paris, vibrated too slowly in Cayenne; he had to shorten it by a line in order to make it again beat seconds accurately. On his return to Paris he had to lengthen the pendulum again by the same amount, since it now went too fast. Newton perceived that this apparently insignificant fact was really of the highest importance, for he recognised that these different rates of oscillation were due in Paris to the less, and in Cayenne to the greater, distance from the centre of the earth. Cassini's discovery of the notable flattening of the planet Jupiter was an additional proof of the truth of Newton's theory. Yet it was not until the middle of last century that Newton's theory was generally accepted as an irrefragable truth.

(To be continued.)

## THE VARIOUS METHODS OF DETERMINING THE VELOCITY OF SOUND

THE propagation of sound is a question with many bearings in the province of physics, and the researches of physicists in relation to it, though numerous, have left some points still under discussion. It is useful in the view of further inquiry to be furnished with a historical survey of what has been already done, and this is the object of a recent memoir by Dr. H. Benno-Mecklenburg, published in Berlin (a *résumé* of which to the following effect appears in the May number of the *Journal de Physique*).

The author has adopted the following classification of the methods that have been employed for measuring the velocity of sound:—

I. Methods requiring the measurement of a time and a course traversed.

1. Direct measurement of the velocity; the most ancient measurements of this kind were executed by P. Mersenne in 1657, by the Academicians of Florence in 1660,<sup>1</sup> by Walker<sup>2</sup> (in England), in 1698; by Cassini and Huyghens (in France), &c.

2. Method of coincidences, indicated by Bosscha,<sup>3</sup> and employed by Koenig.<sup>4</sup>

<sup>1</sup> Newton, "Philosophia Naturalis Principia Mathematicæ," II., Prop. XLVIII.—L.

<sup>2</sup> Laplace, "Mécanique Céleste," t. v. livre xii. p. 115.

<sup>3</sup> Tentamina, "Exper. Acad. del Cimento," 1738, xi. p. 116.

<sup>4</sup> Philosophical Transactions, 1698.

3. Apparatus of Neumann<sup>1</sup> and Le Roux.<sup>2</sup>

## II. Estimation of the velocity of sound by the number of vibrations and the wave-length of musical sounds.

## A. Direct methods:—

1. Method of Bernoulli, with sonorous tubes.

2. Method of Chladni,<sup>3</sup> with rods.

3. Method of Kundt.

4. Methods of Stefan<sup>4</sup> and Warburg.<sup>5</sup>

## B. Methods based on the interference of sonorous waves:—

1. Method of Savart.<sup>6</sup>2. Method of measurement of the wave-length with Quincke's interference tubes.<sup>7</sup>3. Method of Zach.<sup>8</sup>

4. Method of beats.

The way in which the velocity of sound is affected by certain circumstances, especially intensity and pitch, requires further elucidation. Up till recent times it was believed, in accordance with the earlier observations and the theoretical formulæ of Newton and Laplace, that sound is propagated with a uniform velocity in the same medium, the temperature remaining constant; that the velocity of sound in air at zero, *e.g.*, is an invariable quantity. After an observation by Parry related by Sir James Ross, that the sound of a cannon was always heard sooner than the word of command to fire, Schröder van der Kolck was led by theory to a formula giving the velocity of sound in a gas as a function of the relation of the two specific heats and the degree of compression of the medium. This velocity would be greater the more intense and grave the sound, and would diminish with the distance traversed.

Regnault set himself to determine rigorously the ratio of the two specific heats of gases, with a view to deducing the mechanical equivalent of heat. He remarked that Newton and Laplace had assumed, in their formulæ, that the gases were perfect, *i.e.* (1), that they followed Mariotte's law exactly; (2) that their elasticity was not altered by surrounding bodies; and (3) that gas opposes no inertia to the transmission of sound-waves. Accordingly the propagation of sound was supposed the same whatever the intensity. Regnault's more complete formula indicated that the velocity is greater the greater the intensity of the wave.

Experiment proved that the intensity of the wave diminishes in a tube more rapidly the smaller the section. The wave is weakened by the reaction of the elastic walls of the tube, causing a considerable loss of *vis viva*; and the diminution of intensity, according to Regnault's formula, should result in diminution of velocity, which diminution must be more rapid the narrower the tube. This was confirmed by experiment.

As regards experiments with the human voice and wind-instruments, the following are the principal observations of Regnault; acute sounds are propagated with much less facility than grave sounds; in very wide pipes, it is necessary to sing with a baritone voice in order to be well heard; the fundamental sound is heard before its harmonics, which succeed in order of pitch, and the timbre is thus altered. The velocity was found independent of pressure, as indicated by the formulæ. Lastly, with different gases, the velocities are inversely proportional to the square roots of the densities.

In connection with the foregoing, it is interesting to compare the results that have been obtained by Kundt.<sup>9</sup> The idea of his method was suggested by Chladni's figures. A tube of glass is used about 2 m. long, containing a certain quantity of lycopodium powder (distributed as regularly as possible), and closed at the two ends. You rub the tube longitudinally, so as to produce a sound. The powder is then seen to accumulate at the nodes of vibration, so that the sonorous waves of the gas are, in a way, rendered visible. The distance from one node to the next being half a wave length, suppose that we have twelve in the tube; the length of the tube vibrating transversely, will be the half of a wave-length in the glass. Under such conditions, then, the length of half a wave in the glass is sixteen times the length of half a wave in the air. It will follow that the velocity of sound in the glass is sixteen times that in air. Other gases may be used in the tube, and the velocity of sound similarly found in them.

By a simple modification of the apparatus, this method gives the velocity of sound in a large number of solid bodies, and the

results agree pretty closely with those found by different methods.

But Kundt's method does not give sufficient precision in respect to the delicate questions investigated by Regnault. The wave-lengths measured never go beyond about 45 mm., making the 0.0001386 part of the course traversed by sound in a second; hence an error of 0.1 mm. made in the measurement of a wave-length would lead to an error of  $\frac{1}{11}$  m. in the result sought. With this reserve, Kundt's results may be here noted.

1. The length of sonorous waves, and consequently the velocity of sound, diminishes proportionally to the diameter of the tube, when this is less than a quarter of the length of undulation.

2. In narrow tubes a high sound is transmitted more rapidly than a grave one, and the diminution of the velocity of sound is in inverse ratio to the wave-length.

3. The velocity of sound is independent of the pressure in a wide tube, but increases with it in a narrow one.

It will be seen that these latter results are in contradiction with those found by Regnault.

It may be generally affirmed that every influence which tends to increase the *vis viva* of the molecules of the sonorous medium has an accelerating action on the velocity of sound, and every influence tending to diminish the *vis viva* diminishes also the velocity.

The causes affecting the velocity of sound are (it is shown) various. In an indefinite medium they are:—1. The temperature of the medium; 2. The quantity of foreign substances found in it, *e.g.*, water-vapour; 3. The pitch of the sound; 4. The direction and force of the wind; 5. In solid bodies, the direction of the sound in relation to the molecular structure.

## In sonorous tubes:—

6. The diameter; 7. The curvature; 8. The rugosity of the interior surface; 9. The thickness of the walls.

The action of the following additional causes is still disputed:—

1. The intensity of the sound; 2. The length of course traversed; 3. The substance forming the tubes.

There is complete disagreement between Regnault, Schröder, Kundt, and Seebeck, as regards the influence of the pitch of the sound. Regnault affirmed merely that an acute sound is transmitted more easily, but not more rapidly, than a grave sound; Schröder finds that the velocity diminishes as the acuteness increases; Kundt and Seebeck reach the contrary result. Fresh experiments are required to settle this important question.

## UNIVERSITY AND EDUCATIONAL INTELLIGENCE

A NEW supplemental charter having been granted to the University of London a few months since, on the joint application of the Senate and of Convocation, empowering the Senate to admit women to graduate in its several faculties (Arts, Science, Law, Medicine, and Music), on such conditions as the Senate, with the concurrence of the Home Secretary, should deem expedient, the Senate lost no time in passing a resolution which made all the existing regulations, relating not only to graduation, but also to the various honours and rewards granted at the several examinations open to female as well as to male candidates. This resolution having been now approved by the Home Secretary, female candidates will be admitted forthwith to the matriculation examination; and all such as have already passed the general examination for women will be considered as having matriculated, and will be admissible (after the required interval) to the first degree examination in either of the faculties. Further, with a view to the special encouragement of female candidates desiring to go through a regular academical course, the trustees of the Gilchrist Educational Trust have instituted two exhibitions, one of 30*l.*, the other of 20*l.*, per annum, tenable for two years, to the female candidates who pass highest in the honours division at the matriculation examination; and two exhibitions, one of 40*l.*, the other of 30*l.*, per annum, tenable for two years, to the female candidates who pass highest at the first B.A. examination (provided that they obtain in the first case two-thirds, and in the second three-fifths, of the total number of marks), to assist them in pursuing their studies at some collegiate institution approved by the trustees; with the further reward of a gold medal of the value of 20*l.* (or of a book-prize of the same value) to the female candidate who passes highest at the second B.A. examination, if she obtains not less than two-thirds of the total number of marks. These rewards are quite independent of those granted

<sup>1</sup> Pogg. Ann., t. xcii. p. 485.<sup>3</sup> Pogg. Ann., t. cxxviii. p. 307.<sup>4</sup> Comptes Rendus, t. lv. p. 662.<sup>5</sup> Chladni's "Acoustics."<sup>6</sup> Sitzungsberichte der Wiener Akademie, t. lvii. pp. 197 and 708.<sup>7</sup> Pogg. Ann., t. cxxvii. p. 285.<sup>8</sup> Comptes Rendus, t. vii. p. 1068.<sup>9</sup> Pogg. Ann., t. v. p. 496.